Indian Statistical Institute, Bangalore
B. Math.
First Year, Second Semester
Linear Algebra-II

Back paper Examination
Maximum marks: 100

Date : 8 June 2022
Time: 10.00AM-1.00PM
Instructor: B V Rajarama Bhat

Note: Consider standard inner product on $\mathbb{R}^n$ and $\mathbb{C}^n$ unless some other inner product has been specified.

(1) Consider a block matrix
$$P = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$$
where A, D are square matrices. Show that: (i) $\det(P) = \det(A).\det(D)$. (ii) If P is invertible, write down a formula for the inverse of P and prove it. [15]

(2) State and prove Cauchy-Schwarz inequality for complex inner product spaces. [15]

(3) Show through an example that Gram-Schmidt process of three linearly independent vectors taken in different orders say, v_1, v_2, v_3 and v_3, v_2, v_1 can give rise to different sets of ortho-normal vectors. [15]

(4) Suppose M is a subspace of a finite dimensional inner product space V. Show that
$$(M^{\perp})^{\perp} = M.$$
[15]

(5) Write down spectral decomposition, polar decomposition and singular value decomposition of following matrices.
$$A_1 = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$
[15]

(6) Find the values of $x \in \mathbb{R}$ for which the $n \times n$ matrix $B = [b_{ij}]_{1 \leq i,j \leq n}$ given by
$$b_{ij} = \begin{cases} 1 & \text{if } i = j; \\ x & \text{if } i \neq j. \end{cases}$$
is positive. [15]

(7) Suppose D is a non-zero 3×3 matrix such that the minimal polynomial of D is given by $q(x) = x^3 + x^2 - 6x$. (i) Determine the eigenvalues of D. (ii) Show that D is diagonalizable. (iii) Write down the inverse of $I - D$ as a polynomial in D.
[15]